Post-Schooling Human Capital Investments and the Life Cycle of Earnings

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We propose an original model of human capital investments after leaving school in which individuals differ in their initial human capital obtained at school, their rates of return and costs of human capital investments, and their terminal values of human capital at an arbitrary date in the future. We derive a tractable reduced-form Mincerian model of log earnings profiles along the life cycle that is written as a linear factor model in which levels, growth, and curvature of earnings profiles are individual specific. This provides a structural interpretation of results obtained in the empirical literature on the dynamics of earnings and acknowledges its limitations.

I. Introduction

Since the seminal work of Friedman and Kuznets (1945), a large empirical literature studying earnings dynamics has emerged (Meghir and

This is a substantially revised version of an earlier paper of ours presented at the First French Econometrics Workshop in 2009. We thank Christian Belzil, Richard Blundell, Laurent Gobillon, Jim Heckman, Bernard Salanié, and three anonymous referees for helpful

Electronically published May 2, 2018
[*Journal of Political Economy*, 2018, vol. 126, no. 3]

Pistaferri 2010). Landmark articles such as Lillard and Willis (1978), Hause (1980), or MaCurdy (1982) introduced parsimonious statistical representations of earnings processes to assess the relative empirical importance of permanent and transitory components using panel data of earnings. The reason for this decomposition builds on the observation that permanent changes in earnings have a greater impact on individual welfare than transitory ones (Blundell 2014). However, not much attention is devoted to its economic underpinnings and how economically interpretable permanent differences between individuals may contribute to explain earnings dispersion and its evolution over the life cycle.

Another more structural strand of the literature focuses on the estimation of human capital investment models derived from Ben-Porath (1967). In particular, this model provides a rationale for earnings equations à la Mincer at the price of certain approximations and an ad hoc linear assumption on the decline of the investment rate with experience. These equations are widely used by labor economists to account for the effect of education and experience on earnings over the life cycle. Their most standard specification takes the form of the logarithm of earnings being written as a quadratic function of experience to capture the curvature of earnings profiles. Education affects the level of earnings and does not interact directly with experience. This specification has been very successful because of its simplicity and its ability to account for the main empirical features of earnings profiles in many different contexts (Polachek 2008).

In this paper, we propose a post-schooling human capital investment model inspired by Ben-Porath (1967), which results in a linear factor model for life cycle profiles of individual earnings as those pioneered by Hause (1980), Carneiro, Hansen, and Heckman (2003), or Aakvik, Heckman, and Vytlacil (2005). Our baseline factor model of earnings has three factors constituting a level term, a linear trend, and an exponential term that captures the curvature of earnings profiles. Interestingly, associated factor loadings are functions of four individual-specific parameters that have an economic interpretation. First, agents differ in their returns to investments; that is, some are more productive in transforming invested time in productive skills. Second, we assume that the marginal utility cost of invested time is heterogeneous within the population. Third, we allow the terminal value of human capital to vary across individuals and infer from the curvature of the

comments and discussions and seminar participants at Yale, Tilburg, Toulouse, CREST, Pompeu Fabra, Leuven University, Panel Data 2012 in Paris, the European meeting of the Econometric Society 2012 in Malaga, Jean-Pierre Florens's Festschrift in Toulouse, Duke, Hong Kong University, McGill, Society of Labor Economists—European Association of Labour Economists 2013, Ecole Polytechnique, University College of London, Mannheim, the Cowles conference 2014, and Allied Social Science Associations 2015 for their helpful comments. This research has received financial support from the European Research Council under the European Community's Seventh Framework Program FP7/2007–13 grant agreement 295298. All errors remain ours.

earnings profile the implicit horizon of investment that agents consider. This follows Lillard and Reville (1999), who insist on this crucial aspect of earnings dynamics. Finally, initial human capital levels when agents enter the labor market are taken as given.

In a sense, we are extending to post-schooling investments described by Mincer (1974) what was developed some time ago for schooling investments in human capital (see the surveys by Card [2001] and Heckman, Lochner, and Todd [2006]). Moreover, while schooling is viewed as a limit case of our setup, working full-time is possible in contrast to the Ben-Porath model. Investments in human capital can stop well before the terminal period, and this justifies in an exact way the flat spot condition under which human capital prices can be estimated (Heckman, Lochner, and Taber 1998; Bowlus and Robinson 2012).

The linear factor representation that we propose is appealing for the following reasons. First, it provides a bridge between structural models of human capital investments and models of earnings dynamics. It sheds light on the structural interpretation of specifications used in the literature on the dynamics of earnings (see Meghir and Pistaferri [2010] for a survey) such as the heterogeneous income processes and restricted income profiles examined by Baker (1997) or Guvenen (2009). Our model encompasses both of them and unravels their underlying restrictions on the heterogeneity of structural parameters.

Second, the linear representation makes clearer the conditions for parameter identification that is difficult to prove in more complicated models in which parametric specifications are often assumed in order to make the estimation tractable. Specifically, using panel data on earnings only, one structural parameter, namely the terminal value of human capital, is point-identified while the others are partially identified only. Additional data allow all structural parameters to be identified.

Third, the estimation of this structural model is simpler than alternatives such as the nonlinear earnings equation estimated by Polachek, Das, and Thamma-Apiroam (2015), in which nonlinearities and the ensuing incidental parameter issue are difficult to deal with. Our linear setup is also fully compatible with the view that human capital stocks are perfectly substitutable within education or skill groups while they are imperfect substitutes between groups as discussed in Browning, Hansen, and Heckman (1999). Finally, the analytical expressions of the earnings equation and investment profiles are very convenient in structural estimation.

Adopting a highly stylized human capital model comes at the price of simplifying other elements. We treat search and job mobility as frictions under the form of exogenous shocks (see, e.g., Postel-Vinay and Turon 2010) that contribute to the transitory part of the income process. We neglect nonproportional taxes and we adopt a partial equilibrium setup in contrast with Heckman et al. (1998). These authors were among the first to estimate a human capital investment model at school and later in life in a dynamic and stochastic general equilibrium setup that allows the effect of skill-biased technical change on inequality to be estimated. Guvenen and Kuruscu (2012) analyze as well an equilibrium setup with heterogeneous agents investing in human capital. Huggett, Ventura, and Yaron (2011) use such a microeconomic model calibrated with Panel Study of Income Dynamics data to decompose inequality into their long-run individual determinants and short-run shocks. In these papers, unobserved heterogeneity in the human capital production technology needs to be much more restricted than in our approach.

Moreover, individual-specific parameters are assumed to be known by the agents in contrast to the studies of Cunha, Heckman, and Navarro (2005) or Guvenen (2007). From an empirical perspective, some reduced-form specifications, such as that of Browning, Erjnaes, and Alvarez (2012), are richer in terms of heterogeneity because they also estimate other characteristics of the earnings distribution. Polachek et al. (2015) derive a nonlinear approximation obtained by the truncation of a series from Haley's (1976) human capital investment model. Their parameters are individual specific, and they describe these heterogeneity terms as functions of cognitive ability, personality traits, and family background.

Another restriction of our baseline framework is that the linear factor representation of the earnings equation is obtained by assuming away consumption smoothing or labor supply flexibility. We provide a more elaborate structural model in which consumption is smoothed at the price of the linear factor model representation that can no longer be justified. Furthermore, we prove that allowing the two channels of intertemporal smoothing through human and financial capital to be opened would intertwine human capital investments and current and future savings, and the level of savings would affect the earnings equation. On the basis of this enriched framework, we provide estimating equations for earnings and consumption when richer data are available.

In the next section, we present the model of human capital accumulation and derive the predicted life cycle profile of earnings. In Section III, we state the economic and econometric restrictions that yield a linear factor model of life cycle earnings and we analyze the identification conditions. Section IV extends this analysis with richer data and in particular earnings and consumption panel data. Section V presents conclusions.

II. The Model

We present an original model of human capital investment in discrete time sharing common features with Ben-Porath (1967) but not all. Specifically, we characterize the optimal sequence of post-schooling human capital investments over the life cycle of agents who maximize their utility over their lifetime. Agents start with an individual-specific level of human capital obtained at school and have individual-specific costs, individual-specific rates of return for investments, and individual-specific terminal values of human capital stocks. Our structural assumptions on the decision problem lead to a closed-form solution for the life cycle profile of earnings whose dynamics depend on individual-specific abilities to earn and to learn (Browning et al. 1999).

A. *The Setup*

Individuals enter the labor market at period $t = 1$, and time and potential experience are confounded. Schooling and the entry decision in the labor market are considered as given. We follow Heckman et al. (1998) by assuming that the post-schooling human capital production process differs from the one affecting school investments, although both are interdependent. Schooling as the main element of previous human capital accumulation and as a determinant of labor market entry is likely to be correlated with individual-specific characteristics affecting post-schooling investments in human capital.

From period 1 onward, agents can acquire human capital by devoting time or effort to training. Human capital is assumed to be of one type only, skills are general, and costs are borne by the workers. Labor supply is inelastic, and potential individual earnings, $y_i^P(t)$, is the product of the individual-specific stock of human capital, $H_i(t)$, by its individual-specific price, $\exp(\delta_i(t))$, which yields $y_i^P(t) = \exp(\delta_i(t))H_i(t)$. Individuals face uncertainty through the variability of human capital (log) prices $\delta_i(t)$, which are mainly affected by aggregate shocks but also by individual ones when there are frictions (e.g., search, information asymmetry, or learning as in Rubinstein and Weiss [2006]). Firms might temporarily value individual-specific human capital in a way that differs from the market in order to attract, retain, or discourage specific individuals, or because information is imperfect. The human capital (log) price, $\delta_i(t)$, is a realization of a stochastic process and is fully revealed at period t to the agent. We do not provide a market analysis of the wage equilibrium process and take it as given (in terms of its distribution).[1]

Current individual earnings are assumed to be given by

$$y_i(t) = \exp(\delta_i(t))H_i(t)\exp(-\tau_i(t)), \tag{1}$$

in which $1 - \exp(-\tau_i(t))$ can be interpreted as the fraction of working time or, alternatively, the fraction of working effort, devoted to investing in human capital. This fraction is increasing in $\tau_i(t)$, equal to zero when

[1] We defer the presentation of the stochastic properties of random processes until Sec. III and of their statistical properties until Sec. III.B.

$\tau_i(t) = 0$, and equal to one when $\tau_i(t) = +\infty$; in this sense, full-time learning is a limit case. We call $\tau_i(t) \geq 0$ the level of investment in human capital at time t.

The technology of production of human capital is described by

$$H_i(t + 1) = H_i(t) \exp[\rho_i \tau_i(t) - \lambda_i(t)], \quad (2)$$

in which $H_i(t)$ is the stock of human capital, ρ_i is the individual-specific rate of return of human capital investments, and $\lambda_i(t)$ is the depreciation of human capital in period t. Depreciation $\lambda_i(t)$ embeds individual-specific or aggregate shocks that depreciate previous vintages of human capital. Individual-specific shocks can be negative because of unemployment periods or of layoffs followed by mobility across sectors. These shocks can also be positive when certain components of human capital acquire more value because of voluntary moves across firms or sectors. As the (log) price $\delta_i(t)$, the variable $\lambda_i(t)$ is assumed to be revealed at period t to the agent, and we treat the distribution of $\lambda_i(t)$ as given.

The human capital technology differs from that of Ben-Porath (1967) in two ways. First, returns ρ_i to investments are constant in the level of human capital, $H_i(t)$.[2] Second and more importantly, agents could stop investing in human capital before the end of the horizon in contrast to Ben-Porath's study, in which full-time working is a limit case. Indeed in that paper, returns to investments $\tau_i(t)$ are equal to $+\infty$ at $\tau_i(t) = 0$ and investments are by consequence never equal to zero. Another consequence is that the last marginal unit of investment today is infinitely less productive than the first marginal unit of investment tomorrow. Equalizing marginal productivities of investments today and tomorrow is what uniquely determines investments.

Our model relies on a different rationale. Investments are as productive today and tomorrow, and the agent decides to stop investing or learning today because effort is costly in utility terms, as specified below. Agents can stop investing before the end of the horizon because costs are too high, and this justifies in an exact way the notion of "flat spots" that Heckman et al. (1998) have proposed as an approximation in an otherwise standard Ben-Porath model. Further considerations on the importance of flat spots for identification are discussed in Section III.B.

The next step is to formulate a utility flow and the way individuals move assets across time. In order to generate the popular log-linear specification for the earnings equation (e.g., Mincer 1974), we assume that period t utility is equal to current log earnings net of investment costs and that there is no consumption smoothing over time. We investigate the conse-

[2] The proof that an extension to nonconstant returns leads to a more general factor model is presented in Magnac, Pistolesi, and Roux (2013).

quences of dropping the latter assumption in Section II.D. Period t utility is written as

$$\ln y_i(t) - c_i \frac{\tau_i(t)^2}{2},$$

in which the cost of investment in utility terms is individual specific and quadratic. We neglect the linear component of the cost in terms of $\tau_i(t)$ because it cannot be identified as current log earnings are derived from equation (1):

$$\ln y_i(t) = \delta_i(t) + \ln H_i(t) - \tau_i(t),$$

and the unit in which $\tau_i(t)$ is expressed is unobserved. Increasing marginal costs fits well with the interpretation of $\tau_i(t)$ as an exerted effort that decreases current earnings and provides future returns. This is what makes unique the solution $\tau_i(t)$ in the dynamic programming.

The decision program of individuals maximizing their discounted expected utility stream over the present and future is given by the following Bellman equation:

$$V_t(H_i(t), \tau_i(t)) = \delta_i(t) + \ln H_i(t) - \left[\tau_i(t) + c_i \frac{\tau_i(t)^2}{2}\right] + \beta_i E_t[W_{t+1}(H_i(t+1))], \qquad (3)$$

in which β_i is the individual-specific discount rate and

$$W_{t+1}(H_i(t+1)) = \max_{\tau_i(t+1)} V_{t+1}(H_i(t+1), \tau_i(t+1)).$$

The terminal condition of this decision program could be written by specifying an individual-specific date at which investing in human capital stops as in Ben-Porath (1967). We proceed differently by using the dual formulation that the value of human capital stocks at an arbitrary date in the future is individual specific.[3] This specification avoids the "regression to the mean" emphasized by Huggett et al. (2011) that would make individual profiles closer and closer at the end of the working life.

Specifically, the value function at the future date $T+1$ or the discounted value of utility stream from $T+1$ onward is written as

$$W_{T+1}(H_i(T+1)) = \delta_i^* + \kappa_i \ln H_i(T+1). \qquad (4)$$

In this expression, κ_i can be interpreted as the capitalized value of one consumption unit over the remaining periods of life after $T+1$ and

$$\kappa_i = 1 + \beta_{i,T+2}[1 + \beta_{i,T+3}(1 + \cdots)],$$

[3] This could be the last date of observation in an empirical analysis.

in which discount rates $\beta_{i,t}$ vary with period t and embody heterogeneous survival probabilities after $T + 1$. If we assume that discount factors $\beta_{i,t>T+1} \leq \beta_i$, for example, $\beta_{i,t>T+1} = \beta_i \Pr(\text{Survival at } t)$, then for all i,

$$\kappa_i \leq \frac{1}{1 - \beta_i}. \tag{5}$$

This suggests that a general interpretation of period $T + 1$ is as a separating date between a span of periods before T in which the probability of survival is equal to one and a span of periods after $T + 1$ in which the survival probability is less than one. As human capital investments are embodied, a smaller discount rate is a source of decreasing returns to investment as the original argument by Mincer put it, and this explains the concavity of earnings profiles.

In summary, investments are driven by individual-specific parameters describing abilities of agents to earn and to learn. The initial human capital level at time $t = 1$ is an ability to earn parameter while returns to investments, ρ_i, and costs of learning, c_i, describe the ability to learn since both affect the accumulation of human capital. Parameter κ_i is the implicit value that individuals place on human capital at horizon T and, as such, can also be considered as an ability to earn parameter, although such an interpretation is less straightforward.

B. *Investment Profiles*

As time t denotes the time elapsed since labor market entry or potential experience, we call the sequence of investments between $t = 1$ and $t = T$ a life cycle profile of investments. When human capital investments are always positive, this profile is summarized in the following proposition.

Proposition 1. Suppose that

$$\beta_i \rho_i \kappa_i > 1. \tag{6}$$

Then

$$\tau_i(t) = \frac{1}{c_i}\left\{\rho_i\left[\frac{\beta_i}{1 - \beta_i} + \beta_i^{T+1-t}\left(\kappa_i - \frac{1}{1 - \beta_i}\right)\right] - 1\right\} > 0 \tag{7}$$

for all $t \leq T$.

Proof. See Section A in the appendix.

Equation (7) expresses the well-known result that human capital investments decrease with time. The term in β_i^{-t} indeed means that it is always better to invest earlier than later because the horizon over which investments are valuable is becoming shorter and shorter (Becker 1964; Mincer 1974; Lillard and Reville 1999). This is the negative value of $\kappa_i - [1/(1 - \beta_i)]$ (condition [5]) that commands the intensity of the decrease.

In addition, levels of investments increase with returns, ρ_i, and decrease with costs, c_i.

Condition (6) ensures that investments in human capital are positive until period T. Nonetheless, investments could stop before period T. Because investments are decreasing, the absence of investments in a period t, $\tau_i(t) = 0$, means that no investments would take place later on, $\tau_i(t') = 0$ for all $t' \geq t$. In consequence, we can proceed backward and analyze the conditions under which human capital investments stop before the last period.

PROPOSITION 2. There exists an optimal stopping period for human capital investments denoted $T_i \in \{1, ..., T+1\}$ such that for all $t \geq T_i$, $t \leq T$, $\tau_i(t) = 0$, and $\tau_i(T_i - 1) > 0$, if and only if

$$\frac{1}{\kappa_{i,T_i}} < \beta_i \rho_i \leq \frac{1}{\kappa_{i,T_i+1}}, \tag{8}$$

where $\kappa_{i,T+1} = \kappa_i$ and $\kappa_{it} = 1 + \beta_i \kappa_{i,t+1}$ for all $t \leq T$ (and by convention $1/\kappa_{i,T+2} = +\infty$ and $1/\kappa_{i,1} = 0$). Additionally, for all $1 \leq t < T_i \leq T+1$, investments are given by

$$\tau_i(t) = \frac{1}{c_i}\left\{\rho_i\left[\frac{\beta_i}{1-\beta_i} + \beta_i^{T_i - t}\left(\kappa_{i,T_i} - \frac{1}{1-\beta_i}\right)\right] - 1\right\} > 0 \quad \forall\ t < T_i. \tag{9}$$

Proof. See Section B of the appendix.

Even if life cycle investments can stop at period $T_i \leq T+1$, the shape of the profile before this period remains similar. This proposition also shows that with information about the stopping time of human capital investments, we could tie in this information with parameters ρ_i and κ_i. The cost parameter, c_i, does not affect the duration of investments but their level only. This is a strong prediction of our setup because costs do not depend on human capital stocks.

C. *The Life Cycle Profile of Earnings*

We deduce from the investment profile the life cycle profile of earnings.

PROPOSITION 3. If T_i is the optimal stopping period defined in proposition 2, log earnings are

$$\ln y_i(t) = \eta_{i1} + \eta_{i2} t + \eta_{i3} \beta_i^{-t} + v_{it} \quad \text{if} \quad t < T_i, \tag{10}$$

$$\ln y_i(t) = \ln y_i(T_i) + v_{it} - v_{iT_i} \quad \text{if} \quad t \geq T_i, \tag{11}$$

in which

$$\eta_{i1} = \ln H_i(1) - \frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\frac{\beta_i^{T+2}}{1-\beta_i} - \frac{\rho_i + 1}{c_i}\left(\rho_i \frac{\beta_i}{1-\beta_i} - 1\right), \tag{12}$$

$$\eta_{i2} = \frac{\rho_i^2}{c_i}\frac{\beta_i}{1-\beta_i} - \frac{\rho_i}{c_i}, \tag{13}$$

$$\eta_{i3} = \frac{\rho_i}{c_i}\beta_i^{T+1}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\left(\rho_i\frac{\beta_i}{1-\beta_i} - 1\right), \tag{14}$$

and v_{it} is defined by

$$v_{it} = \delta_i(t) - \sum_{l=1}^{t-1}\lambda_i(l) = \delta_i(t) - \Lambda_i(t). \tag{15}$$

Proof. See Section C of the appendix.

Proposition 3 shows that the life cycle profile of earnings can be decomposed sequentially into a first span of periods in which human capital investments are positive and the earnings equation (10) has a nonlinear factor structure and a second span of periods in which investments stopped and earnings are a function of price and depreciation shocks only.

During the first span of periods, earnings given by (10) are the sum of a deterministic component and a stochastic one. The first component is fully deterministic for the agent because it depends on individual-specific parameters, η_{i1}, η_{i2}, η_{i3}, β_i, and potential experience, t, only. Furthermore, the reduced-form parameters η_{i1}, η_{i2}, and η_{i3} are functions of the deep parameters $H_i(1)$, ρ_i, c_i, κ_i, and β_i. First, the level of log earnings η_{i1} is affected one to one by the initial human capital stock, $H_i(1)$, with correction factors. Second, the individual-specific growth rate η_{i2} depends positively on the return ρ_i and negatively on the cost c_i. Finally, parameter η_{i3} (which depends on $\kappa_i - [1/(1-\beta_i)]$) and the discount rate β_i control the degree of curvature of the profile and the effect of the horizon of investment. The closer to zero parameter η_{i3} is or the closer to one β_i is, the less curved the profile is.

The stochastic term v_{it} in earnings equation (10) as defined in equation (15) is the (log) price of human capital, $\delta_i(t)$ net of the cumulative human capital (log) depreciations, $\Lambda_i(t) = \Sigma_{l=1}^{t-1}\lambda_i(l)$, since labor market entry. We will refer to it thereafter as the net (log) price of human capital. Components $(\delta_i(t), \lambda_i(t)) \equiv \zeta_i(t)$ are the sources of stochastic dynamics that affect earnings. Not much structure is needed in the dynamic model on these stochastic components except that they are not under the control of the agent. The developments in this section and the proofs in the appendix are valid under general independence assumptions such as[4]

[4] We shall make additional technical assumptions such as $E_{t-h}(|\delta_i(t)|) < \infty$ and $E_{t-h}(|\lambda_i(t)|) < \infty$ that make the dynamic program well defined. For the sake of readability these standard assumptions are not fully stated here (see Stokey and Lucas 1989).

$$\zeta_i(t) \perp (H_i(t), ..., H_i(2))|\zeta_i(t-1), ..., \zeta_i(1), \rho_i, c_i, \kappa_i, \beta_i, H_i(1).$$

In this sense, the derivations of the model above are robust to quite general assumptions on the expectational side of the model as seen from the proofs in the appendix.

D. *Consumption Smoothing and Flexible Labor Supply*

We now return to the assumption that consumption is not smoothed over time using financial assets. Allowing for consumption smoothing would open a second channel of intertemporal transfers through financial assets in addition to human capital accumulation. As shown in Section D of the appendix, the investment equation (7) would include an additional simple function of the savings rate. This is also true for the induction equation determining the relative value of human capital κ_{it}.

PROPOSITION 4. Denoting $s_i(t) = [y_i(t) - C_i(t)]/y_i(t)$, the savings rate, we have

$$\tau_i(t) = \frac{1}{c_i}\left[\beta_i\rho_i\kappa_{it+1} - \frac{1}{1 - s_i(t)}\right],$$

in which the sequence $(\kappa_{it})_{t=1,...,T+1}$ is given by $\kappa_{iT+1} = \kappa_i$ and

$$\kappa_{it} = E_{t-1}\left[\frac{1}{1 - s_i(t)} + \beta_i\kappa_{it+1}\right].$$

Proof. See Section D of the appendix.

When current income is less (respectively greater) than consumption, human capital investments would be larger (respectively smaller) than in the absence of consumption smoothing holding the relative value of human capital, κ_{it}, fixed. This illustrates the reaction of investments to a change in their opportunity costs (Browning et al. 1999) and indicates that in periods of low income (with respect to permanent income as measured by consumption) investments are larger. If we now relax that κ_{it} is fixed, individuals who most of the time save ($s_i(t') > 0$ for all $t' > t$) are also those for whom κ_{it} is larger holding the terminal value κ_i fixed. Savings and human capital investments are unsurprisingly complements.

This investment equation can be used in an empirical application as shown in Section IV. Allowing for consumption smoothing, however, breaks the factor structure for earnings since we could not find any specification allowing for consumption smoothing and ensuring that the earnings dynamics equation takes a linear factor format in experience as shown in the next section. Our conjecture is that there does not exist a dynamic model with financial and human capital accumulation that would generate a log earnings equation of the type we find. In other words, the micro-founded factor model for log earnings that we derive next and embeds most equations used in the literature about earnings dynamics is not robust to the presence of consumption smoothing.

Flexible labor supply is another departure from this model that could be entertained (Blinder and Weiss 1976). Here again, this would lead to another policy function as in proposition 4, but the interaction in utility between labor supply and investment, $\tau_i(t)$, would break the factor structure described in equation (10) that makes the results in the literature on the dynamics of earnings interpretable. A fully heterogeneous model integrating consumption, labor supply, and on-the-job learning extending Blundell et al. (2016) would also be much more involved.

We now return to the baseline setting in which consumption tracks income exactly as could be justified by the evidence gathered by Thurow (1969) or Caroll and Summers (1991), and we suppose that labor supply is inflexible. Restrictive assumptions on the discount rate and on random shocks are needed to end up with a linear factor model and to identify the parameters of the deterministic component in the earnings equation (10). These are the issues that we analyze next.

III. Panel of Earnings: Restrictions and Identification

In this section, we investigate how panel data on earnings used in the empirical literature on earnings dynamics (e.g., Panel Study of Income Dynamics, Social Security data, etc.; see Meghir and Pistaferri 2010) can be exploited to identify reduced-form and structural parameters. Without loss of generality, we consider, as in the theoretical model, a single cohort of agents who enter the labor market at the same time and face the same economic environment since this analysis can be replicated for every cohort.

We also assume that investments in human capital remain strictly positive all over the observation period from $t = 1$ to T (i.e., $T < T_i$) so that earnings are given by equation (10). If this is not the case, the life cycle profile of earnings would be the mixture of two different processes: (1) a generalized heterogeneous growth model driven by human capital investments in periods before T and affected by the dynamics of human capital prices net of depreciation and frictions (eq. [10]) and (2) a process driven by the dynamics of human capital prices and frictions only (eq. [11]). Identification would have to rely on specific distributional assumptions about the deep structural parameters.

We first detail how equation (10) can be restricted to get a linear factor model of earnings dynamics. We also state identifying restrictions on the random process of log prices of human capital net of its depreciation, v_{it}, that would be needed for the identification of factor loadings.

Furthermore, we derive the economic structural restrictions that bear on factor loadings in the linear factor model and study the identification of structural parameters. Finally, we review the implications of this setup for extant models of earnings dynamics.

A. A Linear Factor Specification

In equation (10), log earnings are the sum of a deterministic component and a stochastic component, and we start, in this subsection, by restricting the former component. This component is a nonlinear function of experience t and of parameters η_{i1}, η_{i2}, η_{i3}, and β_i.

We could estimate these individual-specific parameters as in Polachek et al. (2015) using orthogonality conditions for v_{it} and nonlinear methods. We do not pursue this path here because we believe that the estimation of such nonlinear expressions at the individual level is fragile and sometimes difficult to achieve.[5]

The robust estimation of individual discount rates requires more information from the data than the typical earnings data set can supply (see, e.g., in an experimental setting, Andersen et al. [2008]) or requires additional restrictions in an observational setup (see, e.g., Alan and Browning 2010) or both (see Sec. IV). Indeed, the expression of the deterministic component of earnings in equation (10) could be approximated by a Taylor expansion of $\ln \beta_i$ around its population average, denoted $\ln \beta$:

$$\eta_{i1} + \eta_{i2} t + \eta_{i3} \beta^{-t} + \eta_{i4} t \beta^{-t},$$

in which $\eta_{i4} = -\eta_{i3}(\ln \beta_i - \ln \beta)$. Identification of η_{i4}, however, would rely on the interaction between a linear trend and a curvature term and is likely to be fragile. This is why we assume in the rest of the section that the discount rate β_i is homogeneous.

Under this assumption and as long as human capital investments remain strictly positive, the (log) earnings equation (10) can be written as a linear factor model,

$$\ln y_{it} = \eta_{i1} + \eta_{i2} t + \eta_{i3} \beta^{-t} + v_{it}, \tag{16}$$

in which the three factors are $f_t = (1, t, \beta^{-t})$ and η_{i1}, η_{i2}, and η_{i3} are individual-specific effects, or factor loadings, defined by setting $\beta_i = \beta$ in proposition 3.

[5] In particular, when the number of observations for each individual is limited. Polachek et al. (2015) report that their estimation method did not converge for around 3 percent of the individuals.

The identifying restrictions on the stochastic component v_{it} that would allow the reduced-form factor loadings η_{i1}, η_{i2}, and η_{i3} to be identified are presented next.

B. Human Capital Prices and Depreciation Rate

We specify statistical restrictions on the stochastic net (log) price of human capital v_{it} in the earnings equation (10), and we propose guidelines for the estimation of the reduced-form parameters of the linear factor model using panel data on earnings. Because of the rich heterogeneity that we allow for, we assume that the number of time periods over which earnings are observed is large enough.

We first decompose v_{it} into aggregate components and individual-specific components. Namely, human capital stocks owned by specific groups in the working population are assumed to be imperfect substitutes in the aggregate production function of the economy while perfect substitution holds within groups. This is a heterogeneous labor force setting in which a general equilibrium analysis can be conducted in the manner of Heckman et al. (1998) to analyze the effect of aggregate shocks, a point we will return to in Section IV. Agents are grouped according to skills, cohorts, and possibly other constant productive characteristics. Aggregate components in v_{it} can then be interpreted as market prices net of depreciation for these types of human capital. In contrast, individual-specific components are interpreted as individual-specific frictions or depreciations. The mechanisms that underlie the specific dynamics of aggregate and individual-specific components are allowed to differ and are left unrelated.

1. Aggregate Components

At the aggregate level of human capital groups, equation (16) can be linearly aggregated into

$$\overline{\ln y_{gt}} = \bar{\eta}_{g1} + \bar{\eta}_{g2} t + \bar{\eta}_{g3} \beta^{-t} + v_{gt}, \tag{17}$$

in which g denotes a group of perfectly substitutable human capital stocks, $\bar{\eta}_{gk} = E(\eta_{ik} | i \in g)$ for $k = 1$, 2, or 3, and $v_{gt} = E(v_{it} | i \in g)$. The terms $\bar{\eta}_{gk}$, $k = 1, 2, 3$, are the aggregate or mean factor loadings for individuals of group g. The term v_{gt} stands for the market log prices of human capital of group g at time t since macro shocks in log prices, $\delta_i(t)$, and depreciation, $\Lambda_i(t)$, are its underlying components. There are no constraints across groups in these dynamics, and they depend on the supply of each group and the possibly changing aggregate production function.

Assume now that we can write

$$v_{gt} = \pi_{gt} + \varepsilon_{gt},$$

in which π_{gt} is a measure of the net log price of human capital in group g at time t and ε_{gt} is a measurement error such that

$$E\left(\varepsilon_{gt} | f_t = (1, t, \beta^{-t})\right) = 0. \tag{18}$$

The availability of a set of prices of human capital, π_{gt}, and condition (18) provide the key restrictions that separate quantities from prices of human capital. Specifically, Heckman et al. (1998) and Bowlus and Robinson (2012) use a "flat spot" condition whereby π_{gt} can be constructed using a subsample of earners who have stopped investing in human capital. Those agents are observed over a window of periods close to the end of their working life (around 50) at which investments have stopped so that their earnings reflect net human capital prices only. The common justification was that investments in Ben-Porath's model are close to zero at the end of the working life. In our setting, a thorough justification is given by equation (11) that describes what the earnings process is after human capital investments stop. There are other measures of π_{gt} using various earnings deflators, for instance, those based on a direct evaluation of labor productivity of groups.

The identification of parameters in equation (17) then proceeds by deflating aggregate log earnings by indices π_{gt} and by using restriction (18). Parameters $\bar{\eta}_{g1}$, $\bar{\eta}_{g2}$, and $\bar{\eta}_{g3}$ can then be recovered by regressing deflated aggregate log earnings on the set of factors $(1, t, \beta^{-t})$ in each group.

2. Individual-Specific Components

Turning to the within-group dimension, we define centered individual factor loadings by their deviations from their means, $\eta^c_{ik} = \eta_{ik} - \bar{\eta}_{gk}$, for $k = 1, 2$, or 3, and $v^c_{it} = v_{it} - v_{gt}$. The earnings equation becomes

$$u_{it} = \ln y_{it} - \overline{\ln y}_{gt} = \eta^c_{i1} + \eta^c_{i2} t + \eta^c_{i3} \beta^{-t} + v^c_{it}, \tag{19}$$

in which u_{it} is the deviation of individual log earnings from their group averages ($\overline{\ln y}_{gt}$). Individual-specific deviations, v^c_{it}, stand for frictions in a model of search and mobility. Indeed what Postel-Vinay and Turon (2010) nicely exposit is that the dynamics of the earnings process is partly controlled by two other processes, which are individual productivity in the current match and outside offers that the agent receives while on the job. In this setting, three things can happen: either earnings remain in the band within the two bounds defined by these processes; earnings are equal to the productivity process because adverse shocks on that process make employee and employer renegotiate the wage contract; or, alternatively, labor earnings are equal to the outside offer in case the employee can either renegotiate with his employer or take the outside offer if productivity is lower that the outside option.

We do not model these frictions and posit that they are mean independent of factors and factor loadings:

$$E(v^c_{it}|f_t = (1, t, \beta^{-t}), \eta^c_i) = 0. \tag{20}$$

Under this restriction, estimates of factor loadings can be recovered by using equation (19) for each individual. It accommodates in particular health shocks that affect depreciation at a random time.

Nonetheless, condition (20) requires that if the depreciation rate has a fixed component, λ_i, because individual i's specific skills depreciate more rapidly, it has to be homogeneous within group g. Otherwise, the individual deviation of prices would exhibit a linear trend with a slope equal to $\lambda_i - \lambda_g$, since

$$\Lambda_i(t) - \Lambda_g(t) \propto \sum_{l=1}^{t-1}(\lambda_i - \lambda_g) = (\lambda_i - \lambda_g)(t-1).$$

This would modify equation (13) relating the growth effect η_{i2} to the structural parameters. However, it would not necessarily affect the linearity of the factor model, but it would invalidate our structural interpretation of factor loadings in terms of returns, costs, and terminal values that we pursue in the next subsection.

C. *Testable Restrictions and Structural Deep Parameters*

We consider from now on that the reduced-form factor loadings (η_{i1}, η_{i2}, η_{i3}) are identified under the restrictions stated in the two previous subsections. Their estimates are obtained by summing aggregate estimates of $\bar{\eta}_g$, derived in Section III.B.1, and individual centered estimates of η^c_i, derived in Section III.B.2.

The structural model imposes not only a linear factor structure on the reduced form but also restrictions on these reduced-form factor loadings. Furthermore, we have assumed that investments in human capital are positive until the last period of observation to get a linear factor representation of the earnings profile:

$$\tau_i(t) > 0 \quad \text{for all} \;\; t \leq T, \tag{21}$$

so that the econometric model is given by equation (16). This subsection also shows that condition (21) is testable.

The nonlinear system of three equations (12), (13), and (14) have four unknown deep parameters, $\ln H_i(1)$, ρ_i, c_i, and κ_i, that are by consequence underidentified although structural restrictions are binding. First, there is no restriction on η_{i1} since equation (12) is the only source of identification of the level of initial human capital $\ln H_i(1)$. Second, structural restrictions consist in statements about the terminal value κ_i or about costs and returns, that is

$$\kappa_i \in \left[0, \frac{1}{1-\beta}\right], \quad c_i > 0, \rho_i > 0, \tag{22}$$

as well as condition (21). Their implications for the reduced form and the identification of structural parameters are summarized in the following proposition.

PROPOSITION 5. Condition (21) and structural restrictions (22) imply the following restrictions on the individual factor loadings η_{i2} and η_{i3}:

$$\eta_{i2} > 0, \quad \frac{\eta_{i3}}{\eta_{i2}} \in \left[-\frac{\beta^{T+1}}{1-\beta}, 0\right].$$

Parameter κ_i is identified and

$$\kappa_i = \frac{1}{1-\beta} + \beta^{-(T+1)} \frac{\eta_{i3}}{\eta_{i2}}.$$

Furthermore, parameters (ρ_i, c_i) are partially identified in the sense that there exist values (ρ_i^L, c_i^L) such that

$$\rho_i \geq \rho_i^L, \quad c_i \geq c_i^L,$$

and a one-to-one relationship

$$c_i = c(\rho_i, \eta_{i2}).$$

Proof. See Section E of the appendix.

These results are intuitive. The growth parameter η_{i2} is positive because human capital investments are productive and the curvature term η_{i3} is negative because the horizon is finite and profiles are concave. It is also this curvature relative to the growth term, and therefore the implicit horizon over which investments are valued, that identifies the capitalized value of future returns to human capital after period $T + 1$.

One point is in order about the partial identification of parameters c_i and ρ_i. Only a function of them is identified from the reduced-form parameters (η_{i2}, η_{i3}). Yet, we do care about their separate identification because the two parameters have different economic consequences. In some counterfactuals, the two parameters may have different impacts so that the counterfactual effect is also partially identified. Nonetheless, the availability of additional data on exogenously varying discount rates, due to mortality for instance, achieves full identification as detailed in Section IV.

D. *Implications for Earnings Dynamics*

Our setup can deliver the well-known predictions of a human capital setting (Ben-Porath 1967) when considered before the optimal stopping period. Structural restrictions make earnings profiles increasing and concave, and the latter reflects the shortening of the investment horizon. Second, the variance of earnings is U-shaped along the life cycle because high-return investors have a steeper earnings profile than low-return individuals experiencing a flatter profile and these profiles cross after a few years (Mincer 1974). Third, because investments in human capital are more intensive at the beginning of the life cycle for the high-return investors, the cross-section correlation, at the beginning of the life cycle, between earnings growth and level, is negative, although this correlation increases along the life cycle and becomes positive (Rubinstein and Weiss 2006).

A brief comparison of our model with the empirical literature on earnings dynamics is also useful. This literature aims at fitting the empirical covariance structure of (log) earnings over the life cycle, that is, u_{it} in our notation, using competing specifications like the one described as heterogeneous income profiles (HIP) or restricted income profiles (RIP; Meghir and Pistaferri 2010). Up to now, there is no consensus in the literature about which specification fits the data best because tests have low power (see Baker 1997; Guvenen 2007; Hryshko 2012). Our linear factor structure embeds both models since the permanent component includes individual-specific levels and growth rates of earnings as HIP does, and the stochastic component can be any mixture of permanent and transitory shocks as in RIP.

This embedding shows how restrictive HIP and RIP models are in terms of heterogeneity. On the one hand, RIP imposes that parameters η_{i2} and η_{i3} are homogeneous, which renders homogeneous the terminal marginal value of human capital stocks using proposition 5. This proposition also shows that RIP is also inconsistent with a model in which either returns to human capital investments or costs are heterogeneous. All the dynamic variation in RIP is given by the processes of prices and depreciations at the individual level and thus is very restrictive in terms of individual heterogeneity entering the structural model.

On the other hand, a curvature effect is absent in HIP, and this model also implicitly imposes that the terminal marginal value of human capital stocks is homogeneous, $\kappa_i = 1/(1 - \beta)$ under the very specific form that all individual horizons are infinite. On top of these restrictions, our three-factor structure might affect the key identifying assumption of HIP versus RIP restricting the correlations between first differences of within shocks (e.g., Blundell 2014) because of the presence of the geometric term.

In our setting, as long as investments in human capital are positive (i.e., $t < T_i$), both heterogeneous income profiles and general random processes are present. Yet, as discussed previously, when human capital investments stop, the model would no longer be a linear factor model since the deterministic individual-specific terms vanish and RIP would prevail. This requires modeling this structural break, and neither our estimation method presented in this section nor HIP or RIP deals satisfactorily with this issue.

IV. Identification Using Richer Data

In the previous section, we show that panel data on earnings are not informative enough to point-identify parameters governing human capital accumulation. This is why we extend the previous identification analysis to cases in which additional data would be available. The first set consists in having exogenously varying discount rates that permit point identification of the rates of return, ρ_i, and costs, c_i. Furthermore, panel data on earnings supplemented with consumption over a long period of time would be key in relaxing the absence of consumption smoothing but also in identifying heterogeneous discount rates. Finally, we detail the type of aggregate data that would be needed to analyze the response to aggregate shocks of earnings, employment, and consumption at the aggregate level in one or several countries.

Varying discount rates.—Additional data on mortality would allow one to point-identify structural parameters ρ_i and c_i. To make this point in a simple way, split the life cycle from $t = 1$ to T into two subperiods from $t = 1$ to T_s and from $T_s + 1$ to T and assume that the mortality rate is equal to zero in the first subperiod and to an observed constant m in the second one. The discount rates in the two subperiods are, respectively, equal to $\beta^{(1)} = \beta$ and $\beta^{(2)} = \beta(1 - m)$. Furthermore, under the restrictions of Section III and using linear factor models adapted to the differences in the discount rates, $\beta^{(j)}$, the reduced-form parameters in each subperiod, $\eta_i^{(1)}$ and $\eta_i^{(2)}$, are identified. Their relationships with structural parameters are still given by equations (12)–(14), which should be appropriately adapted to the existing differences in the terminal values of human capital stocks, κ_{i,T_s+1} and $\kappa_{i,T+1}$.

First, write equation (13) for the two subperiods as

$$\eta_{i2}^{(1)} = \frac{(\rho_i)^2}{c_i}\frac{\beta^{(1)}}{1-\beta^{(1)}} - \frac{\rho_i}{c_i}; \quad \eta_{i2}^{(2)} = \frac{(\rho_i)^2}{c_i}\frac{\beta^{(2)}}{1-\beta^{(2)}} - \frac{\rho_i}{c_i},$$

and note that if $\beta^{(1)} \neq \beta^{(2)}$ and both values are known, these two equations identify $(\rho_i)^2/c_i$ and ρ_i/c_i and therefore (ρ_i, c_i). Note also that the relationship between parameters κ_{i,T_s+1} and $\kappa_{i,T+1}$ given by the induction equation stated in proposition 2 and their expression as a function of the ratio between η_{i3} and η_{i2} given by proposition 5 becomes a testable restriction.

This setting could be easily extended to an empirical model in which mortality rates would evolve over time. Nonetheless, this presumes that mortality rates are not individual specific and that discount rates are not heterogeneous. This is the point we now turn to by showing that additional data on consumption would help in dealing with heterogeneous discount factors.

Consumption and earnings data.—Suppose now that the panel of earnings is completed with data on consumption over the same time span. We shall use the setting of Section II.D and the results of proposition 4 to derive estimating equations. There are three important consequences for estimation. First, the earnings equation is no longer a linear factor model, although the human capital investment equation of proposition 4 can now be used to derive a first-difference equation for log earnings as a function of structural parameters. Second, values of human capital stocks at each period, κ_{it}, are themselves functions of (observed) future savings. Third, the Euler equation for consumption identifies the heterogeneous discount rate.

Section F in the appendix indeed shows the following proposition.

PROPOSITION 6. If v_{it} defined in equation (15) is such that $E_{t-1}\Delta v_{it+1} = 0$, the structural equations for earnings and consumption are

$$E_{t-1}[\Delta \log y_i(t+1)] = \frac{1}{c_i} E_{t-1}\left\{\frac{\rho_i\beta_i(\rho_i+1)+1}{1-s_i(t+1)} - \frac{\rho_i+1}{1-s_i(t)} + \rho_i\beta_i(\beta_i(\rho_i+1)-1)\left[\sum_{l=2}^{T-t}(\beta_i)^{l-2}\frac{1}{1-s_i(t+l)} + \beta_i^{T-t}\kappa_i\right]\right\}, \tag{23}$$

$$E_t\left[\frac{1}{C_i(t+1)}\right] = \beta_i(1+r(t))\frac{1}{C_i(t)}. \tag{24}$$

These conditional moments restrictions can be transformed into estimating equations by using instruments in the information set at period $t-1$ or t such as past earnings and consumption levels.[6] Note that, in conjunction with information on varying discount rates as above, these rich data allow

[6] A similar proposition would apply if E_{t-1} was replaced with E_{t-h} with $h > 1$ so that serial correlation of a finite order in v_{it} can be accommodated.

the full set of heterogeneous coefficients to be point-identified: rates of return, costs, terminal values of human capital stocks, and discount rates.

If data on hours of work are available, introducing flexible labor supply in this framework in a simple way requires two conditions. First, there should be no on-the-job learning in the sense of Imai and Keane (2004), which would add a third channel of intertemporal substitution. Second, there should be no fixed costs of participation, and nonparticipation should be governed by the same parameters as hours of work since with fixed costs dynamic complications arise (Blundell, Magnac, and Meghir 1997). If these two conditions are satisfied, then the marginal rate of substitution between consumption and leisure is proportional to wages or opportunity costs of time. The standard static model of labor supply can be used provided that selection issues due to nonparticipation are dealt with (Killingsworth 1984).

Aggregate data.—Another direction for empirical research would be to revisit the general equilibrium results of Heckman et al. (1998) using this setup. Our developments in Section III can be readily adapted to the case in which series of log earnings aggregated by skills are available in one country or several countries. Complemented with data on employment, capital, savings, and aggregate production, production functions could be estimated and their estimates could feed in a general equilibrium analysis of the effect of aggregate shocks on human capital investments. However, proposition 6 makes clear that the enriched model is nonlinear,[7] and the delicate issue of aggregation would have to be solved in this case.

V. Conclusion

In this paper, we propose a structural model of human capital investments that predicts that earnings profiles over at least part of the working life are given by a linear factor model in which factor loadings are functions of structural parameters. This provides an economic interpretation of parameters estimated in the literature on the dynamics of earnings. We also provide identifying restrictions that allow factor loadings to be estimated using panel data, and we analyze the identification of structural parameters. In a companion paper (Magnac, Pistolesi, and Roux 2014) we use a long panel on a single cohort of private-sector wage earners in France from 1977 to 2007 and we implement the analysis that we detailed in Section III.

Our model is versatile enough to accommodate consumption smoothing. A linear factor model, however, cannot be easily justified if con-

[7] Because of the interactions between individual coefficients and savings rates.

sumption is smoothed or labor supply is flexible. This is why we provide a complementary analysis when richer data are available and in particular how point identification of key parameters can be achieved. Estimating this model under these general conditions might help further understanding the role that human capital investments play in welfare inequalities when measured using consumption profiles. Many other extensions are worth exploring in future research. First, human capital investment profiles seem to vary widely across different education groups. In particular, a pending conjecture would be that investments by the low-skill group stop much earlier than those by the high-skill group. Second, it would be interesting to extend this work to a multisector framework in which human capital investments would have different rates of return. This would allow for partial on-the-job learning by doing, which is absent in this model.

Appendix

Proofs of Propositions and Extensions

A. *Proof of Proposition 1*

The first-order condition of the maximization problem for $t < T + 1$ is

$$-[1 + c_i\tau_i(t)] + \beta_i\rho_i H_i(t+1)E_t\left[\frac{\partial W_{t+1}}{\partial H_i(t+1)}\right] = 0. \tag{A1}$$

The marginal value of human capital is the derivative of the Bellman equation so that by the envelope theorem,

$$\frac{\partial W_t}{\partial H_i(t)} = \frac{1}{H_i(t)} + \beta_i E_t\left[\frac{\partial W_{t+1}}{\partial H_i(t+1)}\right]\frac{H_i(t+1)}{H_i(t)}. \tag{A2}$$

For $t = T + 1$, condition (A2) writes more simply as

$$\frac{\partial W_{T+1}}{\partial H_i(T+1)} = \frac{\kappa_i}{H_i(T+1)} \Rightarrow H_i(T+1)\frac{\partial W_{T+1}}{\partial H_i(T+1)} = \kappa_i,$$

so that, by backward induction, we obtain

$$H_i(T)\frac{\partial W_T}{\partial H_i(T)} = 1 + \beta_i\kappa_i,$$

$$H_i(T-1)\frac{\partial W_{T-1}}{\partial H_i(T-1)} = 1 + \beta_i(1 + \beta_i\kappa_i),$$

and so on. This yields

$$H_i(t+1)\frac{\partial W_{t+1}}{\partial H_i(t+1)} = \frac{1-\beta_i^{T-t}}{1-\beta_i} + \beta_i^{T-t}\kappa_i.$$

Replacing in equation (A1) yields

$$1 + c_i\tau_i(t) = \beta_i\rho_i\left[\frac{1}{1-\beta_i} + \beta_i^{T-t}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\right]$$
$$= \rho_i\left[\frac{\beta_i}{1-\beta_i} + \beta_i^{T+1-t}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\right],$$

and equation (7) follows. Furthermore, as the second term in (A1) is constant, the second-order condition is satisfied if and only if $c_i > 0$.

Furthermore, and given that $c_i > 0$, the condition that investments are always positive yields

$$\rho_i\left[\frac{\beta_i}{1-\beta_i} + \beta_i^{T+1-t}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\right] - 1 \geq 0 \quad \forall\ t < T+1.$$

As $\kappa_i - [1/(1-\beta_i)] < 0$ and $\beta_i < 1$, $\tau_i(t)$ is decreasing in t because of the term β_i^{-t}, and the right-hand side attains its minimum at $t = T$. This yields condition (6) since

$$\rho_i\left[\frac{\beta_i}{1-\beta_i} + \beta_i\left(\kappa_i - \frac{1}{1-\beta_i}\right)\right] - 1 \geq 0 \Leftrightarrow \rho_i \geq \frac{1}{\beta_i\kappa_i}.$$

QED

B. Proof of Proposition 2

First, condition (8) is not empty since

$$\kappa_{it} = 1 + \beta_i\kappa_{i,t+1} > \kappa_{i,t+1} \Leftrightarrow \kappa_{i,t+1} < \frac{1}{1-\beta_i} \Leftrightarrow \kappa_{i,t+2} < \frac{1}{1-\beta_i}$$

and by repetition

$$\Leftrightarrow \kappa_{i,T+1} = \kappa_i < \frac{1}{1-\beta_i},$$

which is equation (5).

We proceed by backward induction. By proposition 1, we know that

$$\tau_i(T) > 0 \Leftrightarrow \frac{1}{\kappa_{i,T+1}} < \beta_i\rho_i \ \leq\ +\infty$$

($= 1/\kappa_{i,T+2}$ by convention) and, under this latter condition, that equation (7) and therefore equation (9) are satisfied for all $t \leq T$.

By backward induction, assume that this property is true until period $t+1$ for some $t+1 \leq T$:

$$\forall\ t' \geq t+2, t' < T+1, \tau_i(t') = 0 \text{ and}$$
$$\tau_i(t+1) > 0 \Leftrightarrow \frac{1}{\kappa_{i,t+2}} < \beta_i\rho_i \ \leq\ \frac{1}{\kappa_{i,t+3}}, \tag{A3}$$

and under this latter condition, that equation (9) is satisfied for all $t' \leq t+1$. As a proof of proposition 2 by backward induction, we thus shall prove that condition (A3) is true at period t and that equation (9) is satisfied.

We analyze separately the condition $\tau_i(t') = 0$ for all $t' \geq t + 1$ and the condition $\tau_i(t) > 0$.

Assume first that $\tau_i(t') = 0$ for all $t' \geq t + 1$ so that the condition $\tau_i(t') > 0$ is violated for any $t' \geq t + 1$, and therefore, by equation (A3), $\beta_i\rho_i \leq 1/\kappa_{i,t+2}$. Conversely, if $\beta_i\rho_i \leq 1/\kappa_{i,t+2}$, then $\tau_i(t') = 0$ for all $t' \geq t + 1$ because equation (A3) is satisfied for $t' \geq t + 1$. Furthermore, conditions $\tau_i(t') = 0$ imply a simple form for the Bellman equation (3),

$$W_t(H_i(t')) = \delta_i(t') + \log H_i(t') + \beta_i E_{t'} W_{t'+1}(H_i(t' + 1)),$$

and the accumulation equation (2),

$$\log H_i(t' + 1) = \log H_i(t') - \lambda_i(t').$$

Using equation (4), where we set $\kappa_{i,T+1} = \kappa_i$, and the linearity of the two previous equations leads to the condition derived by induction again,

$$W_{t'}(H_i(t')) = \delta_i^*(t') + \kappa_{i,t'} \log H_i(t') \tag{A4}$$

for any $t' \geq t + 1$ and where $\kappa_{it} = 1 + \beta_i\kappa_{i,t+1}$.

Second, assume that $\tau_i(t) > 0$. Proposition 1 can be recast in a setup in which the last period becomes $T_i = t + 1$ instead of $T + 1$ since there are no further human capital investments after this date and since the value function can be written as in equation (A4) evaluated at $t' = t + 1$. This yields equation (9). Equation (9) at period t is

$$\tau_i(t) = \frac{1}{c_i}\left\{\rho_i\left[\frac{\beta_i}{1-\beta_i} + \beta_i\left(\kappa_{it+1} - \frac{1}{1-\beta_i}\right)\right] - 1\right\} > 0, \tag{A5}$$

which is equivalent to $\beta_i\rho_i > 1/\kappa_{it+1}$.

Therefore, the equivalence stated in the proposition is true at period t. Furthermore, equation (9) applies for any $t' \leq t$. The statement under induction is true at $t = T + 1$ and is therefore true at any date $t \in \{1, ..., T\}$. By convention we set $1/\kappa_{i0} = 0$ in order to cover all cases since $\rho_i > 0$.

Using the expression $\kappa_{i,t} = 1 + \beta_i\kappa_{i,t+1}$, we obtain by induction that

$$\begin{aligned}\kappa_{i,t} &= \frac{1}{1-\beta_i} + \beta_i^{T+1-t}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\\ &= \frac{\beta_i}{1-\beta_i} + \beta_i^{T_i - t}\left(\kappa_{i,T_i} - \frac{1}{1-\beta_i}\right),\end{aligned} \tag{A6}$$

while T_i is defined as

$$\frac{1}{\kappa_{i,T_i}} < \beta_i\rho_i \leq \frac{1}{\kappa_{i,T_i+1}}.$$

QED

C. Proof of Proposition 3

First, the stock of human capital in period t depends on previous investment choices and past depreciation, that is,

$$H_i(t) = H_i(1)\exp\left[\sum_{l=1}^{t-1}\rho_i\tau_i(l) - \sum_{l=1}^{t-1}\lambda_i(l)\right] \quad \text{for } 2 \le t.$$

Using equation (1), we can write the logarithm of observed earnings in period t as

$$\ln y_i(t) = \delta_i(t) + \ln H_i(1) + \sum_{l=1}^{t-1}\rho_i\tau_i(l) - \sum_{l=1}^{t-1}\lambda_i(l) - \tau_i(t). \tag{A7}$$

If $t < T_i$, insert the structural expression for $\tau_i(\cdot)$ given by equation (9) of proposition 2 into the first sum of equation (A7) to get

$$\sum_{l=1}^{t-1}\rho_i\tau_i(l) = \frac{\rho_i^2}{c_i}\sum_{l=1}^{t-1}\left[\frac{\beta_i}{1-\beta_i} + \beta_i^{T_i-l}\left(\kappa_{i,T_i} - \frac{1}{1-\beta_i}\right)\right] - \frac{\rho_i}{c_i}(t-1).$$

Because of equation (A6), this sum is equal to

$$\begin{aligned}
&\frac{\rho_i^2}{c_i}\sum_{l=1}^{t-1}\left[\frac{\beta_i}{1-\beta_i} + \beta_i^{T+1-l}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\right] - \frac{\rho_i}{c_i}(t-1)\\
&\quad = \frac{\rho_i^2}{c_i}\frac{\beta_i}{1-\beta_i}(t-1) + \frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\beta_i^T\sum_{l=1}^{t-1}\beta_i^{1-l} - \frac{\rho_i}{c_i}(t-1)\\
&\quad = \left(\frac{\rho_i^2}{c_i}\frac{\beta_i}{1-\beta_i} - \frac{\rho_i}{c_i}\right)(t-1) + \frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\beta_i^T\frac{1-(1/\beta_i)^{t-1}}{1-1/\beta_i}\\
&\quad = -\frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\frac{\beta_i^{T+1}}{1-\beta_i} + \left(\frac{\rho_i^2}{c_i}\frac{\beta_i}{1-\beta_i} - \frac{\rho_i}{c_i}\right)(t-1)\\
&\qquad + \frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\frac{\beta_i^{T+2}}{1-\beta_i}\beta_i^{-t},
\end{aligned}$$

which writes as the sum of three factors whereas one factor is in levels, the second one is a linear trend, and the last one is a geometric trend.

Using equation (9) and equation (A6),

$$\tau_i(t) = \frac{1}{c_i}\left(\rho_i\frac{\beta_i}{1-\beta_i} - 1\right) + \frac{\rho_i}{c_i}\beta_i^{T+1}\left(\kappa_i - \frac{1}{1-\beta_i}\right)\beta_i^{-t},$$

and rearranging expression (A7), we obtain equation (10).

If $t = T_i$, we can derive a similar expression by using $\tau_i(t) = 0$, but it will be of no use in the following.

If $t > T_i$, which in our setting can apply only if $T_i \le T$, investments from period T_i onward are equal to zero. Hence, for $t \ge 2$,

$$\begin{aligned}
H_i(t) &= H_i(1)\exp\left(\sum_{l=1}^{T_i-1}\rho_i\tau_i(l) - \sum_{l=1}^{t-1}\lambda_i(l)\right)\\
&= H_i(1)\exp\left(\sum_{l=1}^{T_i-1}\rho_i\tau_i(l) - \lambda_i(l) - \sum_{l=T_i}^{t-1}\lambda_i(l)\right)\\
&= H_i(T_i)\exp(-\Lambda_i(t) + \Lambda_i(T_i)).
\end{aligned}$$

This gives

$$\begin{aligned}\ln y_i(t) &= \ln H_i(T_i) + \Lambda_i(T_i) + \delta_i(t) - \Lambda_i(t)\\ &= \ln y_i(T_i) + v_{it} - v_{iT_i},\end{aligned}$$

which corresponds to equation (11). QED

D. *Proof of Proposition 4*

Assume that consumption is smoothed over time. The new dynamic program is written as

$$\max_{C_i(t),\tau_i(t)} \left[\log(C_i(t)) - c_i\tau_i(t)^2/2 + \beta_i E_t W_{it+1}(A_i(t+1), H_i(t+1))\right]$$

under the constraints

$$\begin{aligned}A_i(t+1) &= [1 + r_i(t)]A_i(t) + y_i(t) - C_i(t),\\ y_i(t) &= \exp(\delta_i(t))H_i(t)\exp(-\tau_i(t)),\\ H_i(t+1) &= H_i(t)\exp(\rho_i\tau_i(t) - \lambda_i(t)),\end{aligned}$$

where $A_i(t)$ is the stock of assets or debt detained by individual i and $r_i(t)$, the interest rate at date t.

Static first-order conditions write

$$\frac{1}{C_i(t)} - \beta_i E_t \frac{\partial W_{it+1}(A_i(t+1), H_i(t+1))}{\partial A_i(t+1)} = 0,$$

$$-c_i\tau_i(t) + \beta_i E_t \frac{\partial W_{it+1}(A_i(t+1), H_i(t+1))}{\partial A_i(t+1)}[-y_i(t)]$$

$$+ \beta_i E_t \frac{\partial W_{it+1}(A_i(t+1), H_i(t+1))}{\partial H_i(t+1)}\rho_i H_i(t+1) = 0,$$

in which the second term on the second line comes from

$$\frac{\partial A_i(t+1)}{\partial \tau_i(t)} = \frac{\partial A_i(t+1)}{\partial y_i(t)}\frac{\partial y_i(t)}{\partial \tau_i(t)} = -y_i(t).$$

Replacing the first in the second first-order condition yields

$$c_i\tau_i(t) + \frac{y_i(t)}{C_i(t)} = \beta_i E_t \frac{\partial W_{it+1}(A_i(t+1), H_i(t+1)))}{\partial H_i(t+1)}\rho_i H_i(t+1).$$

If $y_i(t) = C_i(t)$, this is condition (A1). Note that

$$\frac{y_i(t)}{C_i(t)} = \frac{1}{1 - s_i(t)},$$

in which $s_i(t)$ is the savings rate, and denote the new marginal relative value of human capital

$$\kappa_{i,t+1} = E_t \frac{\partial W_{it+1}(A_i(t+1), H_i(t+1)))}{\partial H_i(t+1)} H_i(t+1) \tag{A8}$$

so that the equation above yields

$$\tau_i(t) = \frac{1}{c_i}\left[\rho_i\beta_i\kappa_{i,t+1} - \frac{1}{1 - s_i(t)}\right].$$

This replaces equation (A5) in the previous section if investments are positive.

We now turn to the dynamics of $\kappa_{i,t+1}$. Using the envelope theorem, dynamic conditions yield that $[\partial W_{it}(A_i(t), H_i(t))]/\partial H_i(t)$ is equal to

$$\begin{aligned}
&\beta_i\left(\frac{\partial E_t W_{it+1}(A_i(t+1), H_i(t+1))}{\partial A_i(t+1)}\frac{\partial A_i(t+1)}{\partial H_i(t)} + \frac{\partial E_t W_{it+1}(A_i(t+1), H_i(t+1))}{\partial H_i(t+1)}\frac{\partial H_i(t+1)}{\partial H_i(t)}\right)\\
&= \beta_i\left(\frac{\partial E_t W_{it+1}(A_i(t+1), H_i(t+1))}{\partial A_i(t+1)}\frac{y_i(t)}{H_i(t)} + \frac{\partial E_t W_{it+1}(A_i(t+1), H_i(t+1))}{\partial H_i(t+1)}\frac{H_i(t+1)}{H_i(t)}\right)\\
&= \frac{1}{H_i(t)}\frac{y_i(t)}{C_i(t)} + \beta_i\frac{\partial E_t W_{it+1}(A_i(t+1), H_i(t+1))}{\partial H_i(t+1)}\frac{H_i(t+1)}{H_i(t)}.
\end{aligned}$$

At period t, this equation and equation (A8) yield

$$\begin{aligned}
\kappa_{it} &= E_{t-1}\left[H_i(t)\frac{\partial W_{it}}{\partial H_{it}}\right]\\
&= E_{t-1}\left[\frac{1}{1 - s_i(t)} + \beta_i\frac{E_t\partial W_{it+1}(A_i(t+1), H_i(t+1))}{\partial H_i(t+1)}H_i(t+1)\right]\\
&= E_{t-1}\left[\frac{1}{1 - s_i(t)} + \beta_i\kappa_{it+1}\right].
\end{aligned}$$

This relation holds even when $\tau_i(t) = 0$, and here again, if $y_i(t) = C_i(t)$, this is the induction relationship of the previous section, $\kappa_{it} = 1 + \beta_i\kappa_{i,t+1}$ (with no expectation since the relationship is deterministic there). QED

E. *Proof of Proposition 5*

The two equations (13) and (14) simplify to

$$\begin{aligned}
\eta_{i2} &= \frac{\rho_i}{c_i}\left(\rho_i\frac{\beta}{1-\beta} - 1\right),\\
\eta_{i3} &= \frac{\rho_i}{c_i}\beta^{T+1}\left(\kappa_i - \frac{1}{1-\beta}\right)\left(\rho_i\frac{\beta}{1-\beta} - 1\right).
\end{aligned} \tag{A9}$$

Taking the ratio of the second and the first equations yields

$$\frac{\eta_{i3}}{\eta_{i2}} = \beta^{T+1}\left(\kappa_i - \frac{1}{1-\beta}\right).$$

We derive the restriction from $\kappa_i \in [0, 1/(1-\beta)]$ that

$$\frac{\eta_{i3}}{\eta_{i2}} \in \left[-\frac{\beta^{T+1}}{1-\beta}, 0\right]. \tag{A10}$$

Conversely, if this restriction is valid, then κ_i is given by

$$\kappa_i = \frac{1}{1-\beta} + \beta^{-(T+1)}\frac{\eta_{i3}}{\eta_{i2}} \in \left[0, \frac{1}{1-\beta}\right].$$

Furthermore, proposition 1 proved that investments remain positive until period T (inclusively) if and only if $\beta\rho_i\kappa_i > 1$. This yields that

$$\rho_i > \rho_i^L = \frac{1}{\beta\kappa_i} = \frac{1}{[1/(1-\beta)] + \beta^{T+1}(\eta_{i3}/\eta_{i2})} > 0$$

by the above. The first equation of (A9),

$$\eta_{i2} = \frac{\rho_i}{c_i}\left(\rho_i\frac{\beta}{1-\beta} - 1\right) = \frac{\rho_i}{c_i\kappa_i}\left(\frac{\rho_i\beta\kappa_i}{1-\beta} - \kappa_i\right),$$

also implies that, given that all parameters are positive,

$$\eta_{i2} > \frac{\rho_i}{c_i\kappa_i}\left(\frac{1}{1-\beta} - \kappa_i\right) > 0.$$

Conversely, assume that $\eta_{i2} > 0$ and $\rho_i > \rho_i^L$. By construction, the condition $\beta\rho_i\kappa_i > 1$ is satisfied and investments are positive until T. Second, define

$$c_i = \frac{\rho_i}{\eta_{2i}}\left(\rho_i\frac{\beta}{1-\beta} - 1\right),$$

and write

$$\frac{\partial c_i}{\partial \rho_i} = \frac{1}{\eta_{2i}}\left(2\rho_i\frac{\beta}{1-\beta} - 1\right),$$

which is positive since $\rho_i[\beta/(1-\beta)] > 1$ because $\beta\rho_i\kappa_i > 1$ and $\kappa_i \leq 1/(1-\beta)$. Both expressions prove that

$$c(\rho_i, \eta_{2i}) = \frac{\rho_i}{\eta_{2i}}\left(\rho_i\frac{\beta}{1-\beta} - 1\right)$$

is positive and increasing in ρ_i. Therefore, $c_i \geq c_L = c(\rho_L, \eta_{2i})$. QED

F. Dynamic Equations of Earnings and Consumption

When consumption can be smoothed, the earnings equation is no longer given by a linear factor model. Because savings and human capital investments interact, it is easier to write the earnings equation in first differences. Earnings at date t are

$$\log y_i(t) = \log H_i(t) - \tau_i(t) + \delta_i(t)$$

and human capital stock at date $t+1$ is

$$\log H_i(t+1) = \log H_i(t) + \rho_i\tau_i(t) - \lambda_i(t)$$

so that earnings at date $t+1$ are

$$\log y_i(t+1) = \log H_i(t) + \rho_i\tau_i(t) - \lambda_i(t) - \tau_i(t+1) + \delta_i(t+1);$$

and taking first differences to get rid of log $H_i(t)$,

$$\Delta \log y_i(t+1) = (\rho_i + 1)\tau_i(t) - \tau_i(t+1) + \Delta\delta_i(t+1) - \lambda_i(t).$$

The investment equation is derived from proposition 4:

$$\tau_i(t) = \frac{1}{c_i}\left[\rho_i\beta_i\kappa_{it+1} - \frac{1}{1 - s_i(t)}\right],$$

and replacing in the difference of log earnings we get

$$\Delta \log y_i(t+1) = \frac{1}{c_i}\left\{\left[\rho_i\beta_i\kappa_{it+1} - \frac{1}{1 - s_i(t)}\right](\rho_i + 1) - \left[\rho_i\beta_i\kappa_{it+2} - \frac{1}{1 - s_i(t+1)}\right]\right\} + \Delta v_i(t+1),$$

in which $\Delta v_i(t+1)$ is the first difference of the random shocks in log earnings (see proposition 6). By assumption of the proposition, we have that

$$E_{t-1}\Delta v_i(t+1) = 0.$$

Furthermore from proposition 4,

$$\kappa_{it+1} = E_t\left[\frac{1}{1 - s_i(t+1)} + \beta_i\kappa_{it+2}\right], \quad \text{(A11)}$$

and we get for all $t \le T - 1$

$$\begin{aligned} E_{t-1}(\Delta \log y_i(t+1)) &= \frac{1}{c_i}E_{t-1}\left\{\left[\rho_i\beta_i\frac{1}{1 - s_i(t+1)} + \rho_i\beta_i(\beta_i\kappa_{it+2}) - \frac{1}{1 - s_i(t)}\right](\rho_i + 1) - \left[\rho_i\beta_i\kappa_{it+2} - \frac{1}{1 - s_i(t+1)}\right]\right\} \\ &= \frac{1}{c_i}E_{t-1}\left[\frac{\rho_i\beta_i(\rho_i + 1) + 1}{1 - s_i(t+1)} - \frac{\rho_i + 1}{1 - s_i(t)} + \rho_i\beta_i(\beta_i(\rho_i + 1) - 1)\kappa_{it+2}\right], \end{aligned}$$

in which $s_i(t)$ is an observed variable if consumption and income are observed.

Furthermore, savings are also observed until date T, and therefore κ_{it+2} can be expressed as a function of observables, $s_i(t')$, $t' \ge t$, and parameters κ_i, ρ_i, and β_i by using equation (A11):

$$\kappa_{it+2} = E_{t+1}\left[\sum_{l=2}^{T-t}(\beta_i)^{l-2}\frac{1}{1 - s_i(t+l)} + (\beta_i)^{T-t}\kappa_i\right], \quad t \le T - 2.$$

This delivers the first equation in proposition 6 and moment conditions when we interact it with variables in the information set at dates before $t - 1$.

Furthermore, the second equation is the Euler equation:

$$E_t\frac{1}{C_i(t+1)} = \beta_i(1 + r(t))\frac{1}{C_i(t)},$$

which identifies β_i. QED

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